WS 30 Unit 3 Review

1) $f(x) = 7\sqrt{x}$, $x = 4$

point: $f(4) = 14$, $(4, 14)$

slope: $f(x) = 7x^{1/2}$

$f'(x) = \frac{7}{2}x^{-1/2}$

$f'(4) = \frac{7}{2\sqrt{4}} = \frac{7}{4}$

T: $y - 14 = \frac{7}{4}(x-4)$

N: $y - 14 = -\frac{4}{7}(x-4)$

2) $y = \frac{7x-1}{4x+5}$

$\frac{dy}{dx} = \frac{(4x+5)(7) - (7x-1)(4)}{(4x+5)^2}$

$\frac{dy}{dx} = \frac{28x + 35 - 28x + 4}{(4x+5)^2}$

$\frac{dy}{dx} = \frac{39}{(4x+5)^2}$

3) $f(x) = x^3 - 7x^2 + 8x + 2$

$f'(x) = 3x^2 - 14x + 8$

$3x^2 - 14x + 8 = 0$

$(3x-2)(x-4) = 0$

$x = \frac{2}{3}$ $x = 4$

4) $f(x) = (6x^3 - x + 1)(9 - x^6)$

$f'(x) = (6x^3 - x + 1)(-6x^5) + (9 - x^6)(18x^2 - 1)$

5) $f(x) = \frac{-3x^2 + 2x + 3}{(x-2)^2}$, $x = -2$

$f'(x) = \frac{(x-2)^2(-6x+2) - (-3x^2+2x+3)(2(x-2)\cdot 1)}{[(x-2)^2]^2}$

$f'(2) = \frac{(-4)^2(14) - (-12-4+3)(-8)}{16^2}$

$= \frac{16(14) - 104}{16^2} = \frac{120}{256}$

$= \frac{15}{32}$

6) $y = 8x^2 + 4x$

$y' = 16x + 4$

$16x + 4 = -44$

$16x = -48$

$x = -3$

$y(-3) = 60$

$\boxed{(-3, 60)}$

7) $y = (8x^3 + 7)^{3/2}$

$y' = \frac{3}{2}(8x^3+7)^{1/2} \cdot d(8x^3+7)$

$y' = \frac{3}{2}(8x^3+7)^{1/2}(24x^2)$

$y' = 36x^2(8x^3+7)^{1/2}$

8) $f(x) = \frac{1}{(6x+7)^5} = (6x+7)^{-5}$

$f'(x) = -5(6x+7)^{-6}(6)$

$f'(x) = \frac{-30}{(6x+7)^6}$

9) a) $f(x)$ is continuous on $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$

b) $f(x)$ is differentiable on $(-\infty, -3) \cup (-3, -2) \cup (-2, 2) \cup (2, 4) \cup (4, \infty)$

10) $y = f(g(x))$

$y' = f'(g(x)) \cdot g'(x)$

$y'\Big|_{x=4} = f'(g(4)) \cdot g'(4)$

$= f'(2) \cdot \left(\frac{2}{5}\right)$

$= -3\left(\frac{2}{5}\right)$

$= -\frac{6}{5}$

11) $H(x) = f(x) + g(x)$

$H'(x) = f'(x) + g'(x)$

$H'(3) = f'(3) + g'(3)$

$= -3 + 1$

$\boxed{H'(3) = -2}$